

James Dyson Foundation Bursary Outreach Activity Report – Sam Stephenson

Overview:

- My project focussed on the 3D rotation of a bottle cap. To present this at the outreach day I intent to use demonstrations and examples to help the students grasp new concepts of 3D rigid body rotation which may seem unintuitive. The aim is to get as many students fascinated by the peculiar behaviour of 3D rigid body rotation by not only showing them the strange demonstration but having them have hands on experience of the effects. I focussed mainly on the concept of Moment of Inertia, Intermediate Axis Theorem and Gyroscopic Motion.

Introduction:

- The students are year 12 so have covered a lot of translational mechanics but have not really delved into the world of rotational mechanics. The first part of the talk is about looking at transitional systems and applying Newton's 2nd Law and looking at all the different forces acting on a body to make sure everyone is up to scratch.

Moment of Inertia in 2D:

- We then link the known translational description of Newton's 2nd to the unknown rotational description such that there is a common link for it to be understood, we then look at a few examples of where one can apply this.
- Take extra time to explain this new quantity 'I' the Moment of Inertia as this will be completely new, yet very important. Introduce it as a 'rotational mass' and use the roundabout example with putting people in different places on it makes it easier or harder to spin. Can be useful to explain with energy and how if more mass is on the outside, for the same rotation rate more mass is moving faster and thus has more energy, i.e. takes more work to get it up to speed or for the same work will not end up as fast.
- Experiment: 2D example of Moment of Inertia. We have a solid and hollow cylinder of the same outer radius and mass (and thus different density). Pose the question to the group of students which one will roll down a ramp the fastest. Do the demonstration and observe the answer and then ask the students why this is the case. Go back to the roundabout example and explain how because the hollow cylinder has all its mass concentrated at the edges the same forcing results in a slower rotation.

Moment of Inertia in 3D:

- Expand this concept to 3D. Explain how due to different geometries of objects from different perspectives the moment of inertia or 'rotational mass' can change. Make a point of explaining how the magnitude of I must be specified around an axis and I can change for different axes. Limit this to principal axes and through the centre of mass for ease. A really good example is a table tennis racket.
- Experiment: Hand out table tennis rackets and ask the students to visualise it spinning. How many visualised each of the 3 axes? Ask them for the same amount of force which axes will the racket spin fastest and slowest about and to then throw them up and down spinning

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about each axis and put their hypothesis to the test. Once they have the correct conclusion and reasoning ask them if there was anything else they noticed. A lot of experiments that set out to find one thing can often find another thing. If you're lucky someone will notice the face the racket does not rotate smoothly in one direction. If they don't come to this conclusion get them to throw the racket 'red side up' and then try to catch it 'red side up' and they will find they can only catch it 'black side up'

- Talk briefly about the simplified maths (quadratic equation) behind the intermediate axis theorem and say how this comes up in a lot of bodies, a calculator, a phone, etc. Show the video of the wingnut rotating in space.

Gyroscopic Motion

- Now we can move onto the main aspect of the presentation; gyroscopic motion
- Experiment: Bike Wheel demonstration, I like to start with the demonstration as it is seemingly so counterintuitive the students are amazed by it. Spin up a bike wheel and then hand it down by a string attached to one side of the hub. Get the students to try this for themselves and experience it.
- Once we sit back down, we need to relate some known translational equations to their rotational forms, specifically the equation for angular momentum and change in angular momentum. Relating them to translational quantities makes it much easier for the students to grasp and doesn't require going into the vector calculus of it all. One but of important vectoring is the idea of a direction of rotation.
- Experiment: briefly rotate a disc along its central axis and ask how we can give this a direction of rotation, if someone says 'to the left' then point out there are some bits moving to the right, if someone says 'clockwise' show how a change of perspective can lead to it rotating anticlockwise, and then finally talk about the right-hand rule and show how this can work from all perspectives at all points on the body. I find it helpful to say there is no logic to this, it is just a way to define things and if we all know the rules at the start it works well. This rule can apply to all angular quantities, torque, momentum, etc, and this gives us a common direction of where these things are acting/facing. This makes them easier to relate to the directional nature of things we already understand like forces and momentum, we can intuitively conclude the direction of a messes momentum as its 'difficulty to stop' and which way you'd have to push, we can now think of angular momentum in the same way, just with our direction defined using our new rule.
- Looking at a gyroscope when it is spinning it has a lot of angular momentum. Looking at the forces acting on it, it results in a couple. This must change the angular momentum vector; in the same way a force changes the momentum vector. For angular quantities we have already seen we have an added layer of 'directionness' to things, so we can change the angular momentum by changing its direction. Relate this to known examples and draw on the arrows and show that the system will process itself around. When you know the rules of the game, and each one of the rules sits fine, the counterintuitive motion you see sits a little bit better in peoples minds.
- Experiment: Get the students holding the gyroscope and trying to move it and see how it behaves in strange ways, this was one of the most enjoyable parts of the talk. (Note: one student had a broken hand and was not able to hold the gyroscope with two hands, so we did the one handed procession demonstration with her).

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- Finally, relate the above back to the bike wheel and also relate it to conservation. If we rotate the gyroscope then we change the orientation and change the angular momentum, this results in a component straight up, but the system as a whole has not had any external couples, so this must be balanced, the result is you rotate the opposite direction.
- Experiment: Get the swivel table out and have people stand on it and 'drive themselves' around, definitely one of the most enjoyable parts!

See below a screenshot of the slides used in the presentation:

<p>3D Rotating Bodies</p> <p>Sam Stephenson</p>	<p>Newton's 2nd Law $F = ma$</p>	<p>Translational Motion</p> <p>$F = ma$</p>	<p>Translational Motion</p> <p>$F = ma$</p>
<p>Translational Motion & Rotational Motion</p> <p>$F = ma$ $\tau = I\ddot{\theta}$</p>	<p>Rotational Motion</p> <p>$\tau = I\ddot{\theta}$</p>	<p>What is 'I'?</p> <p>$\tau = I\ddot{\theta}$</p> <p>Three wheels (red, yellow, and blue) are mounted on a vertical axis of rotation. The red wheel is the largest and has the most mass. The yellow and blue wheels are smaller and have less mass. They are all rotating at the same angular velocity. The red wheel has the largest moment of inertia, so it has the largest angular momentum. The yellow and blue wheels have smaller moments of inertia, so they have smaller angular momenta. The total angular momentum is the sum of the angular momenta of the three wheels. The red wheel's angular momentum is the largest, so it dominates the total angular momentum. The total angular momentum is conserved, so the red wheel's angular momentum must remain constant. The yellow and blue wheels' angular momenta must change to keep the total angular momentum constant. This means the red wheel must rotate in the opposite direction to the yellow and blue wheels.</p>	<p>'I' in 3D</p>
<p>Try flipping the bat, what do you notice?</p>	<p>Intermediate Axis Theorem</p> <p>$I^2 + k^2 = 0$ If $k^2 > 0$ then there are real solutions (simple harmonic motion) If $k^2 < 0$ then there are imaginary solutions (unstable motion) $k^2 = \frac{I_C^2 - I_A I_B}{I_A I_B}$ So, $k^2 > 0$ and motion is stable if A is biggest or smallest $k^2 < 0$ and motion is unstable if A is in between B & C</p>	<p>Gyroscopic Motion</p>	<p>Bike Wheel Demonstration</p>
<p>Momentum & Angular Momentum</p> <p>$p = mv$ $L = I\dot{\theta}$</p>	<p>Change in Momentum & Angular Momentum</p> <p>$\Delta p = F$ $\Delta L = \tau$</p>	<p>Vectors of Spin</p>	<p>Application of Torque</p> <p>$\Delta L = \tau$</p>
<p>Angular Momentum</p>	<p>Vectors of Spin</p>	<p>Conservation of Angular Momentum</p>	<p>My Project</p>
<p>Thank you</p> <p>I hope you enjoyed!</p>			