Mystery of the Spinning Bottle Cap – Sam Stephenson

My project investigated the motion of a spinning bottle cap. The topic of 3D rotating rigid bodies can be very counterintuitive and often leads to unexpected results such as the peculiar motion of gyroscopic procession. It is best to start by looking at a general case and some simpler systems before jumping straight in.

Many dynamic systems tend to a state of minimum potential energy when in free motion. What this means is that when a system is left to move under the influence of conservative forces (notably, gravity) the system will accelerate and eventually come to rest. The point at which the system comes to rest will be a configuration where the potential energy of the system is at a minimum. A very simple example is rolling a ball down a valley. When at the top of a hill a ball will roll down unless prevented by an external force. When the ball rolls to the bottom it may begin to roll up the other side. As energy is lost to friction the ball will come to rest at the bottom of the valley. The only potential energy in this system is the gravitational potential energy (GPE) which is a function of the relative height of the ball. It is clear that the minimum height of the ball is at the bottom of the valley, and this corresponds to the configuration of minimum potential energy. o

Another intuitive example of this is a mass on a spring. If a mass is attached to a spring which is attached to a fixed point and the mass is dropped, it will oscillate on the string until it eventually comes to rest. The mass will come to rest at a point of minimum total potential energy (the sum of the GPE from the height of the mass, and the elastic potential energy, the EPE, from the extension in the string). The position of the mass can also be found by a force balance, equating the force of the weight of the mass with the force from the extension of the spring. You will unsurprisingly get the same answer either way.

If we next look at a spinning coin. The coin has two stationary configurations, one on its edge which is unstable, but still stationary, and one on its face, which is stable. It is clear to see that when the coin if flat on its face it's centre of mass is lower than on its edge. This, combined with the stability of the two stationary points, means that the coin will invariably land with its face flat as that is the stable configuration with the minimum potential energy.

If we now look at the spinning bottle cap, we see that because of its geometry it has two stable positions of minimum potential energy it could come to rest in. The first, a global minimum, is with the logo face of the bottle cap facing downwards on the surface, the second, a local minimum, is with the logo face of the bottle cap facing upwards. Logically from what we know, we would expect the cap to always land in the global minimum potential energy configuration. However invariably, the bottle cap will tend to come to rest in its local minimum, with the logo face up, if it has sufficient initial energy. My project aimed to gain a better understating as to why this is the case and also what mechanism causes the motion of the bottle cap.

Throughout the project a number of methods were used to investigate the behaviour of the cap. These can be roughly split into three sections: slow motion footage, experimental methods, and modelling and simulation. Each method of investigation has its benefits and drawbacks and, although they are split into sections, the findings and conclusions from each method tend to overlap as one would expect when working towards a common goal.

The slow-motion footage of the cap shows three distinct regimes of motion, an upright regime near the start of the motion where the cap is slightly tilted towards falling logo face down, a horizontal regime at the end of the motion where the cap is oriented logo face up and is very close to the horizontal and is primarily rolling instead of spinning with slip, and a transitional regime between the two. It is clear that the mechanism causing the motion is happening within the transitional regime. Upon investigating caps with different geometries and mass distributions some parameters were observed to have a large influence, such as the position of the centre of mass relative to the contact patch, but every cap resulted in the same end state. It was concluded that the cause of motion was to do with either the cap rolling on the flanged face, and orienting itself in its translation and rotation such that it gained enough momentum to flip itself over, the cap bouncing and leaving the table for periods of its motion, or a similar motion to the hopping hoop. Ultimately, this problem is concerned with the split of potential and kinetic energy in the motion, and the frictional and reaction forces of the contact patch. Experimental methods were key to narrowing the scope of what causes the bottle cap's unique motion. Initial experiments were done with a variety of bottle caps with different geometrical features and inertial properties. The results of these experiments show that although different parameters of bottle cap do affect the motion in terms of how fast the transition occurs or how long the bottle cap spins for, all bodies with bottle cap like geometry end up in the same orientation, that is, logo face up. Also, a disc with an offset centre of mass will exhibit an even 50/50 split of landing logo face up or logo face down. This shows that if the inertial properties of the cap are necessary, they are not sufficient. A model bottle cap was produced for repeatability of experiments due to the deformities caused when removing a bottle cap from a bottle, and also such that there was a reference from which parameters could be adjusted. The cap was rendered in SolidWorks and the inertial properties analysed. The inertia matrix of the cap can be identical to that of a disc because the ratio of the principle moments of inertia of both bodies are equal. Although the bodies would react the same to a given applied torque, due to the geometry of each body the applied torque is different, hence the motion of each does not evolve in the same way. Experiments changing the friction and initial conditions were done to investigate the argument that the cap needs to overcome an energy barrier to fall the way it does. These experiments showed that there was indeed a cut off where, given sufficient energy, the cap would almost always land logo face up and if the initial conditions were such that the cap had less energy it would be more likely to land logo face down. This energy barrier seems to be a function of the surface as well, as the energy barrier observed was much lower for a smooth surface than it was for a rough surface. Finally, modelling and simulation were used to investigate the motion as well. This quickly becomes very involved so a process is outlined as to how this could be used in future work to investigate things further. Initially, a simplified steady state analysis was done which showed that the gyroscopic stabilising force would actually cause the cap to rotate from leaning towards falling logo face up, to upright. This analysis is only steady state for one specific configuration (the cap rotating about a vertical axis containing the centre of mass and the contact patch) and also does not include the frictional forces of the contact patch. Next, a 2D simplified numerical simulation was modelled. The code takes inputs of initial conditions and time steps through the equations of motion of the system to find the motion. A 2D semicircle was found to behave as expected. The outline of how to expand this code to 3D is given and the equations of motion calculated. The overall conclusion is that the motion of the cap is a function of both its inertial properties and its geometric properties. These two factors influence the external torques acting on the body, given that the torques arise due to forces acting through the centre of mass (position determined by the inertial properties) and forces acting through the contact patch (position determined by the geometric properties). The leading theory presented is that due to the friction on the contact patch, due to translational sliding, there is a torque acting on the cap that results in it gaining angular momentum and rolling. This causes the cap to transfer some of its kinetic energy from its initial spin into kinetic energy of the body rolling. This spinning and rolling motion eventually leads to a configuration where the cap has sufficient momentum in the correct direction to raise the centre of mass and flip itself over to land logo face up. Future work includes using the equations of motions derived in this report to numerically time step the system as a 3D simulation. Once this is done one could include various friction models and allow for slip and one could also use the constraint equations derived as a starting point for altering the geometry of the body in question.